Comment on "Restricted curvature model with suppression of extremal height"

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Recently Jeong and Kim [Phys. Rev. E **66**, 051605 (2002)] investigated the scaling properties of equilibrium self-flattening surfaces subject to a restricted curvature constraint. In one dimension (1D), they found numerically that the stationary roughness exponent $\alpha \approx 0.561$ and the window exponent $\delta \approx 0.423$. We present an analytic argument for general self-flattening surfaces in D dimensions, leading to $\alpha = D\alpha_0/(D+\alpha_0)$ and $\delta = D/(D+\alpha_0)$, where α_0 is the roughness exponent for equilibrium surfaces without the self-flattening mechanism. In case of surfaces subject to a restricted curvature constraint, it is known exactly that $\alpha_0 = 3/2$ in 1D, which leads to $\alpha = 3/5$ and $\delta = 2/5$. Small discrepancies between our analytic values and their numerical values may be attributed to finite size effects.

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Fluctuation properties of equilibrium surfaces have been studied extensively for the last few decades [1]. Surface roughness is well documented and classified into a few universality classes. The Edwards-Wilkinson (EW) class is generic and robust for equilibrium surfaces with local surface tension [2]. The EW surfaces can be described by the continuum Langevin-type equation

$$\frac{\partial h(\vec{r},t)}{\partial t} = -\nu \nabla^2 h(\vec{r},t) + \eta(\vec{r},t), \quad (1)$$

where $h(\vec{r},t)$ is the height at site $\vec{r}$ and time t, $\eta(\vec{r},t)$ is an uncorrelated Gaussian noise, and ν represents the strength of local surface tension.

The surface fluctuation width $W(L,t)$, defined as the standard deviation of the surface height $h(\vec{r},t)$ starting from a flat surface of lateral size L, satisfies the dynamic scaling relation

$$W(L,t) = L^{\alpha} f(t/L^{z}), \quad (2)$$

where the scaling function $f(x) \to \text{const}$ for $x \gg 1$ and $f(x) \sim x^{\beta}$ ($\beta = \alpha/z$) for $x \ll 1$ [1,3].

The EW universality class is characterized by the dynamic exponent $z=2$ and the roughness exponent $\alpha = (2-D)/2$ (D: substrate dimension). In the absence of local surface tension ($\nu = 0$), higher-order local suppression terms like $\nabla^{2m} h$ ($m = 2,3,\ldots$) become relevant to determine the scaling properties of surface roughness. In this case, the scaling exponents become $z = 2m$ and $\alpha = (2m-D)/2$.

Recently, Kim, Yoon, and Park [4] introduced a global-type suppression (self-flattening) mechanism which reduces growing (eroding) probability only at the globally highest (lowest) point on the surface. They found that this global-type suppression changes the scaling properties of the EW surfaces: $z \approx 3/2$ and $\alpha = 1/3$ in one dimension (1D) and $z \approx 5/2$ and $\alpha = 0(\log)$ in 2D. The one-dimensional roughness exponent $\alpha = 1/3$ characterizing the stationary surface fluctuations could be derived exactly by mapping the surface evolution model onto the static self-attracting random walk model [5,6].

More recently Jeong and Kim (JK) [7] investigated the effect of the self-flattening mechanism on the so-called restricted curvature (RC) model [8]. The RC model is known to have no local surface tension term ($\nu = 0$) and its dominant suppression term is of the fourth order ($\nabla^4 h$). Accordingly, the ordinary RC model has the scaling exponents $z = 4$ and $\alpha = (4-D)/2$. Using Monte Carlo simulations, JK found for the one-dimensional self-flattening RC model that

$$z = 1.69(5), \quad \alpha = 0.561(5), \quad \text{and} \quad \beta = 0.332(5). \quad (3)$$

Again, the self-flattening mechanism changes the scaling properties of the RC surfaces.

JK also studied the height-height correlation function $G(r)$, defined as the average of the square of height differences at two sites separated by a distance r. They found an extra length scale ξ (window size) where the correlation function starts to saturate. The correlation function in the steady state satisfies the scaling relation

$$G(r) = L^{2\alpha} g(r/\xi) \quad \text{with} \quad \xi \sim L^{\delta}, \quad (4)$$

where the scaling function $g(x) \to \text{const}$ for $x \gg 1$ and $f(x) \sim x^{2\alpha' - \kappa}$ ($\alpha' = \alpha/\delta$) for $x \ll 1$. This type of a crossover scaling, due to the existence of a smaller length scale ($\delta < 1$) compared to the system size, has been previously identified in the so-called even-visiting random walk (EVRW) models (see Eq. (29) in Ref. [9]). In fact, the EVRW model is intimately related to the EW-type surface model with the self-flattening mechanism [4]. JK's numerical estimates for the exponents are

$$\delta \approx 0.423, \quad \kappa \approx 0.868, \quad \text{and} \quad \alpha' = \alpha/\delta \approx 1.33. \quad (5)$$

In this Comment, we present an analytic argument that predicts the values of the scaling exponents α and δ associated with the stationary properties of surface fluctuations. We consider a general equilibrium surface growth model, the stationary roughness exponent of which is known exactly as α_0. The partition function for equilibrium self-flattening surfaces of lateral size L can be written as [4]

$$Z_L(K)=\sum_{\mathcal{C}} e^{-KH(\mathcal{C})}, \tag{6}$$

where the summation is over all possible height configurations $\mathcal{C}$ subject to a given constraint, K is a temperaturelike parameter, and $H(\mathcal{C})$ is the height excursion width (the globally maximum height minus the globally minimum height) for a given configuration $\mathcal{C}$. Global suppression for self-flattening dynamics is simply the metropolis-type evolution algorithm with this partition function to reach the equilibrium [4]. For the EW surfaces, one can take the surface height configurations subject to the restricted solid-on-solid constraint, where the step heights are allowed to take finite values. In the case of the RC surfaces, the local curvature $\nabla^2 h$ is restricted to be finite.

One can decompose the configurational space into sectors with a constant excursion width H. Then, the partition function can be rewritten as

$$Z_L(K)=\int_0^\infty dH\omega_L(H)e^{-KH}, \tag{7}$$

where $\omega_L(H)dH$ is the number of configurations with the height excursion width between H and $H+dH$. One can define $\Omega_L(H)$ as the number of configurations with the height excursion width less than H as

$$\Omega_L(H)=\int_0^H dH'\,\omega(H'), \tag{8}$$

and it is clear that $\Omega_L(\infty)=Z_L(K=0)$.

We estimate $\Omega_L(H)$ as follows. Consider a flat surface between two walls separated by a distance H and parallel to the surface. The surface starts to fluctuate, following its ordinary evolution dynamics. Whenever the surface hits either of the two walls, the number of possible configurations inside the walls reduces by a constant factor, compared to the no-wall case $(H=\infty)$. This entropic reduction can be roughly translated as

$$\Omega_L(H)\approx\Omega_L(\infty)\exp[-aN_c], \tag{9}$$

where N_c is the typical number of contacts between the walls and the surface in the stationary state and a is a positive constant of $O(1)$. In this estimate, the entropic reduction due to each contact is assumed to be uncorrelated. Typically, there will be $O(1)$ contacts over a block of lateral size ℓ, within which the stationary surface width $W_0(\ell)\sim\ell^{\alpha_0}$ is of the same order of magnitude as the wall distance H, i.e.,

$$N_c\sim(L/\ell)^D\sim L^D/H^{D/\alpha_0}. \tag{10}$$

With the above estimates, the partition function for nonzero K becomes

$$Z_L(K)=Z_L(0)\int_0^\infty dH\left(\frac{\partial}{\partial H}e^{-aL^D/H^{D/\alpha_0}}\right)e^{-KH},$$
$$=Z_L(0)K\int_0^\infty dHe^{-KH-aL^D/H^{D/\alpha_0}}. \tag{11}$$

This integral can be evaluated by the saddle point method in the limit of large L. The maximum contribution comes from

$$H^*\sim\left(\frac{L^D}{K}\right)^{\alpha_0/(D+\alpha_0)}. \tag{12}$$

The stationary surface fluctuation width should be proportional to this typical height excursion width, so we predict that

$$W\sim K^{-\alpha_0/(D+\alpha_0)}L^\alpha, \tag{13}$$

with $\alpha=D\alpha_0/(D+\alpha_0)$.

The length scale ℓ arising naturally in our argument should be proportional to the window size ξ in the correlation function. Inside this length scale, the surface does not feel the global self-flattening suppression. As our length scale ℓ is explicitly related to the wall distance in Eq. (10), we can also predict that

$$\ell^*\sim\xi\sim L^\delta, \tag{14}$$

with $\delta=D/(D+\alpha_0)$.

In summary, for the general self-flattening equilibrium surfaces, we predict that

$$\alpha=\frac{D\alpha_0}{D+\alpha_0},\delta=\frac{D}{D+\alpha_0},\quad\text{and}\quad\alpha'=\frac{\alpha}{\delta}=\alpha_0, \tag{15}$$

where α_0 is the stationary roughness exponent for the equilibrium surfaces without the self-flattening mechanism.

For the EW surfaces $\alpha_0=1/2$ in 1D, which lead to $\alpha=1/3$ and $\delta=2/3$. These results agree with those by the other analytic (healing time) arguments [9,10] and those by the numerical simulations [11]. For the RC surfaces $\alpha_0=3/2$ in 1D, which lead to $\alpha=3/5$ and $\delta=2/5$. The short range behavior governing the value of κ should be identical to the ordinary RC model, so we also expect that $\kappa=1$. These results have small discrepancies from the JK's numerical results in Eqs. (3) and (5). We believe that this may be due to rather small sizes used in their numerical simulations. The RC surfaces have $\alpha_0=(4-D)/2$ in D dimensions, so $\alpha=D(4-D)/(4+D)$ and $\delta=2D/(4+D)$. It may be interesting to check these predictions for the RC self-flattening surfaces in $D=2$ and 3.

At the upper critical dimensions ($D=2$ for the EW and $D=4$ for the RC surfaces), the surface roughness becomes logarithmic $(\alpha_0=0)$ and the self-flattening mechanism induces only corrections to scaling in the surface fluctuations. The dominant correction seems to be independent of system size L [4], but its functional dependence on K is not fully explored. In higher dimensions, the EW ($D>2$) and the RC ($D>4$) surfaces are asymptotically flat ($\alpha_0<0$). The self-flattening mechanism induces power-law-type corrections to scaling as given in Eq. (15).

We could not present any analytic explanation for the dynamic exponents z and β. This may be due to the lack of a continuum-type equation to govern the self-flattening mechanism. It would be very interesting to find such an equation which contains a global coupling term in space.

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